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DYNAMIC STABILITY OF SPACE VEHICLES

Volume IV - Full Scale Testing for Flight Control Parameters

by David R. Lukens

Prepared by
GENERAL DYNAMICS CORPORATION
San Diego, Calif.
for George C. Marshall Space Flight Center

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FOREWORD

This report is one of a series in the field of structural dynamics prepared under contract NAS 8-11486. The series of reports is intended to illustrate methods used to determine parameters required for the design and analysis of flight control systems of space vehicles. Below is a complete list of the reports of the series.

Volume I	Lateral Vibration Modes
Volume II	Determination of Longitudinal Vibration Modes
Volume III	Torsional Vibration Modes
Volume IV	Full Scale Testing for Flight Control Parameters
Volume V	Impedence Testing for Flight Control Parameters
Volume VI	Full Scale Dynamic Testing for Mode Determination
Volume VII	The Dynamics of Liquids in Fixed and Moving Containers
Volume VIII	Atmospheric Disturbances that Affect Flight Control Analysis
Volume IX	The Effect of Liftoff Dynamics on Launch Vehicle Stability and Control
Volume X	Exit Stability
Volume XI	Entry Disturbance and Control
Volume XII	Re-entry Vehicle Landing Ability and Control
Volume XIII	Aerodynamic Model Tests for Control Parameters Determination
Volume XIV	Testing for Booster Propellant Sloshing Parameters
Volume XV	Shell Dynamics with Special Applications to Control Problems

The work was conducted under the direction of Clyde D. Baker and George F. McDonough, Aero Astro Dynamics Laboratory, George C. Marshall Space Flight Center. The General Dynamics Convair Program was conducted under the direction of David R. Lukens.

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1/INTRODUCTION

The determination of flight control system parameters and verification of system operation can best be done on a full-scale test stand. The use of component specifications is not adequate to guarantee that the overall flight control system will operate as predicted.

The usual practice is to start the test program as soon as possible in the program. The best time is immediately following the preliminary design of the control system. The testing normally progresses through three distinct phases.

First, a small single-axis stand is built. This stand will usually have a number of its basic parameters such as mass inertia and elastic constraints adjustable. This will allow updating of the model as design progresses. The stand will be used to evaluate the effect of bits and pieces on the overall system performance. The effect of component dynamics on the model used for analysis and simulation will also be continuously updated throughout the program.

Second, testing progresses to the use of specially made test specimens or functional mockups, normally referred to as dynamic test stands. These may include a tail section as well as a stub tank for thrust structure testing, "battleship," or propulsion test models.

Lastly, the testing will progress to flight or flight-type vehicles; normally this phase will emphasize testing with the engines firing. During this portion of the program, flight control testing will constitute only a small portion of the use of the vehicle. This phase will be referred to as static testing (engines firing) in the text.

This monograph on flight control testing is primarily concerned with the major thrust vector control system and its principal components. The types of data that can be determined from full-scale tests are: frequency response, deadbands, friction, nonlinearities, limits, and the effect of structural resonances.

Control system test data are required because the response characteristics of the flight control system are influenced by the total system. Most flight control systems are sufficiently complicated and responsive as to require tests to verify performance.

The underlying basis for all first-of-a-kind launch vehicle flight readiness assertions must be analytical in nature. The only reason that an analytical stability derivation can be used is based on the premise that the analytical model used in analysis and synthesis can be verified by tests. This requires that the flight control system be analyzed for several test configurations and that agreement between model and test be obtained for each configuration. Then and only then can the final design verification be performed with confidence.

This monograph will be limited in scope to testing for the evaluation of the primary thrust vector control system parameters. Combined test stand operation using computer simulation, although another use for the test stand, will not be covered. Other types of test stands are used for complete testing of control systems; these normally use the complete control operating system and allow rotational freedom (1, 2 or 3 axis) by means of a low-friction bearing.

During these tests additional data as to the structural compliance and overall operating environment can be obtained. Duty cycle, expected life cycle, and acceptance level tests are performed using the same setup. The acceptance testing of components and systems, flight readiness, and prelaunch operations may use the same types of programs. However, they are used for completely different purposes and, as such, have different requirements - which are not the primary concern of early test phases.

Combined simulation, component acceptance, environmental, and structural testing - and the determination of life and duty cycles - are not covered in this monograph.

2/STATE OF THE ART

It has always been considered good engineering practice to perform extensive tests on any system before committing it to operational status. Recently the advent of flight vehicles with their attendant large amounts of analysis has tended to cloud the importance of testing. Relegation of the importance of testing came about because a launch vehicle flight control system cannot be ground tested with all flight environments duplicated. In spite of the large amount of analytical studies performed, the adequacy of the flight control system cannot be verified without extensive testing. Thus, bridging the gap between ground and flight conditions becomes the major task of the flight control analyst.

All major vehicle systems contractors maintain and use a number of test beds. These vary from simple single-axis engine gimbal rigs (Reference 1) to complete vehicle systems including structural compliance (Reference 2) to static test sites where it is possible to perform a test of the flight control system with the engines firing (Reference 3).

The types of information gathered for flight control analysis include all the basic values required to define the system. The values of deadband, hysteresis, and linearity limit values are determined as well as the effect of outside influences, parasitic modes, power supply loading, and so forth. In addition to parameter determination, the test stands also provide an excellent test bed for hardware compatibility.

The initial group of measurements consist of gain, linearity, clearance, and such so-called static measurements. All test programs include frequency response testing. This is the most universal of all dynamic testing. The list is usually concluded with tests for friction, stiction, hysteresis, leakage, and other contributors to dead zones.

The use of test stands in combination with computing facilities for flight simulation is also utilized in the launch vehicle development program. These simulations constitute the final per-flight verification of the control system operation versus analysis. This is also the time at which the nominal and dispersion values for flight control components are evaluated. The values to be used for preflight tests, acceptance, and prelaunch go/no-go determinations may also be determined at this time.

3/TEST SETUP

Flight control testing is divided into two general categories. The first, labeled "dynamic testing" includes all tests which do not require firing of the engines. The second, "static testing," includes all configurations where the control system is exercised while the engines are firing.

3.1 DYNAMIC TEST MODELS

The more complete the test model the more faith one will have in the results. Therefore, the best test article will be a complete vehicle. It is often possible to obtain a complete vehicle such as a static test article or production prototype for testing. Then the problems remaining are associated with restraints, tying down of the vehicle, and with supplying power to compensate for those portions of the vehicle power supply which may be inoperative. Because many vehicles are rigidly restrained at the thrust structure during flight and static test it may be necessary to design a special restraint for autopilot testing. This is covered in a separate monograph (Reference 4).

As the elastic restraints imposed upon the thrust structure in flight (free-free) are similar to those which would be imposed if the vehicle were restrained in the center, it is quite satisfactory to use a portion of the vehicle for testing. This setup is graphically illustrated in Figure 1. The use of a well-designed stub tank will yield better results than a mediocre test of a vehicle restrained at the thrust structure. This fact, combined with the convenience of a stub tank, makes it desirable to consider this type of system as part of any flight control test program.

The remaining decisions are discussed in this section. The first question is: should real or dummy engines be used? In the case of Atlas and Saturn, actual engines were used. They had the propellant cooling lines in the bell and the lube oil tank filled with paraffin to duplicate actual firing mass and moment of inertia. If large flexible lines are used it will be desirable to pressurize them to provide proper restraint. In general it will be necessary to provide an external hydraulic or pneumatic power supply. Prototype electronic packages and feedback sensors are usually used if possible. The test signal source may be the same as that used for vehicle checkout or it may be completely different. Specialized recording equipment is usually provided although preflight equipment may also be used or checked out using this type of test setup.

Flight control sensors may be used in a closed loop capacity during these tests, although, in general, gimbaling tests yield more accurate data if run open-loop. It is desirable to have the sensors turned on and to monitor their output for all testing. These types of test setups are ideal for use with computer simulations. They provide an excellent opportunity to check the effect of actual hardware response on the analytic simulation. A typical procedure for hardware substitution into an analog simulation would be:

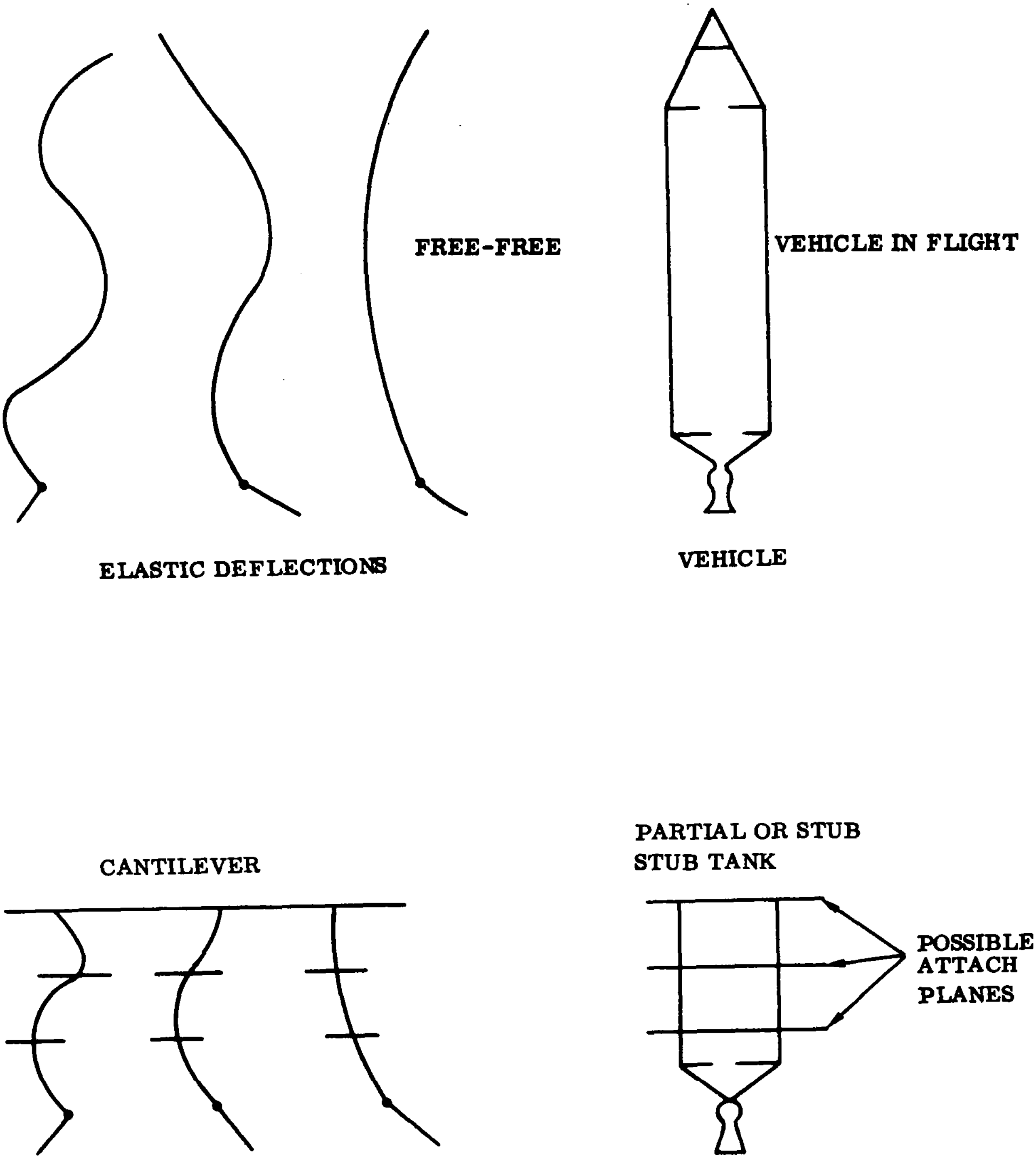


Figure 1. Test Vehicle Setup

a. Use of small electronic components such as amplifiers, filter networks, and logic circuits in the simulation.

b. Use of a single-axis engine gimbal stand in first, fixed-time simulation, and finally, time-varying coefficients.

c. Use of a three-axis test stand as an extension of step b.

There is a fourth step, use of flight control sensors, gyros, accelerometers, etc., mounted on a moveable flight table as part of a combined simulation. This step involves many additional stability and control problems and, as such, is beyond the scope of this monograph.

In addition to tests of a complete flight control engine gimbal system it is also desirable to use simple test fixtures early in the program. A typical gimbal setup would be a single-axis fixture with engine mass and inertia modeled and the mount elasticity simulated. A fixture of this type would usually be designed so as to allow wide variations in the mass, inertial, and elastic properties of the test stand. A stand of this nature is invaluable early in a program. It provides a vehicle for testing of the first breadboard and prototype components, as well as evaluating such items as mount elasticity as the servo system resonates. This type of simulation also provides for early substitution of hardware components into an analog simulation. Small size, compared with the vehicle, and relatively low cost combine to make a single-axis test stand a very important step in a flight control development program.

In addition to the basic test stand the question of instrumentation and equipment must also be resolved on a point-by-point basis when the test stand is assembled. A few guidelines should be noted at this time. The equipment to be installed on the test stand must have its characteristics well known. On most test work the results will be used to verify analytical results.

The basis for flight readiness will be the statement that an analytical model has indeed been obtained for which the results agree with test values. This model has been re-evaluated with flight rather than test parameters and used to verify the final design.

Proper instrumentation is vital to any test program. The instrumentation may vary from providing simple input-output data, Figure 2, up to complete acceleration monitoring of the complete vehicle, Figure 3, the important item being that all variables which could be of interest be monitored. This concept is subject to reasonable hardware limitations. However, it should be remembered that a variable which is not recorded cannot be evaluated. Also, information which is out of band, either in amplitude or frequency, is lost just as surely as if it had not been recorded.

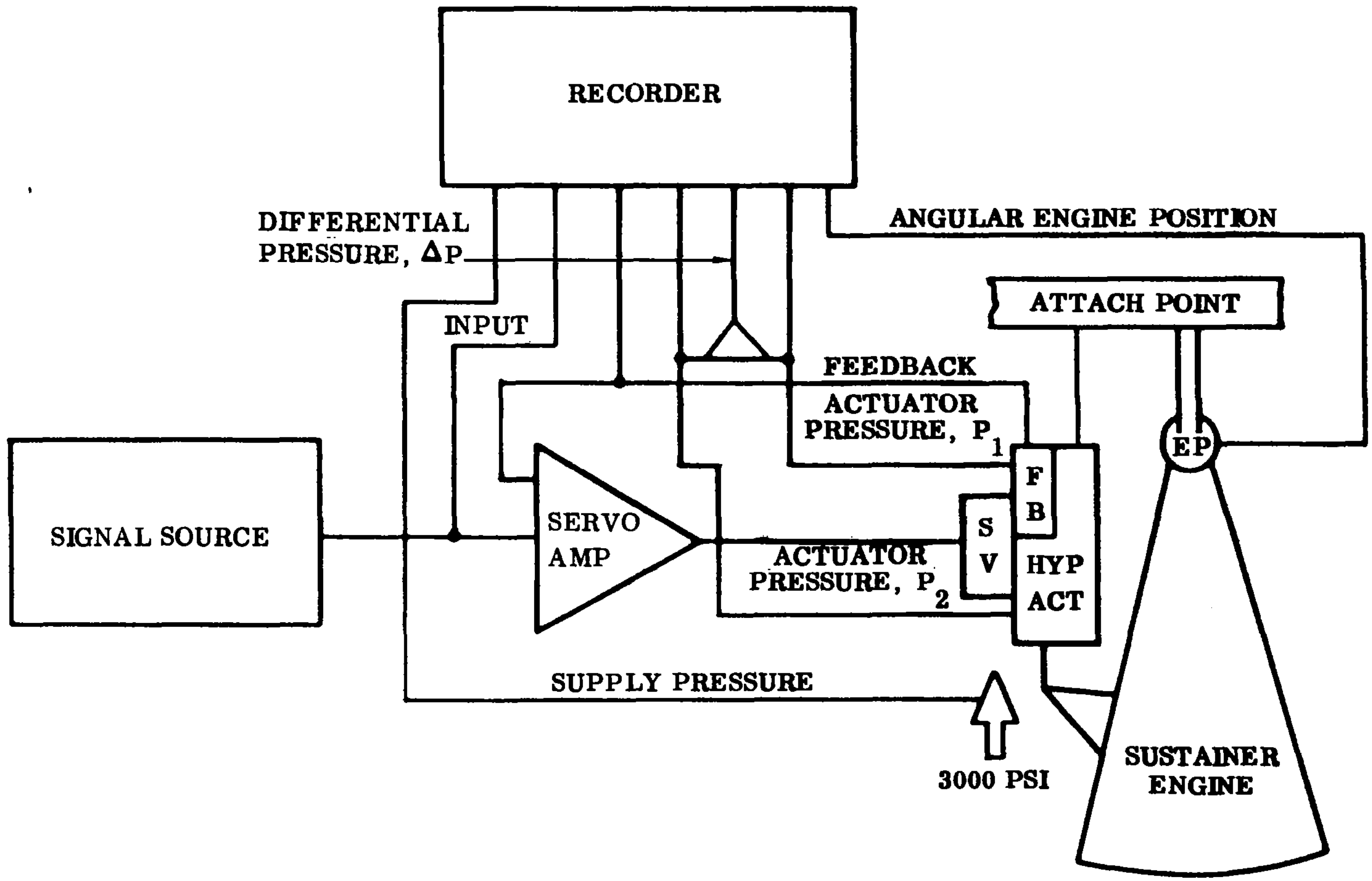
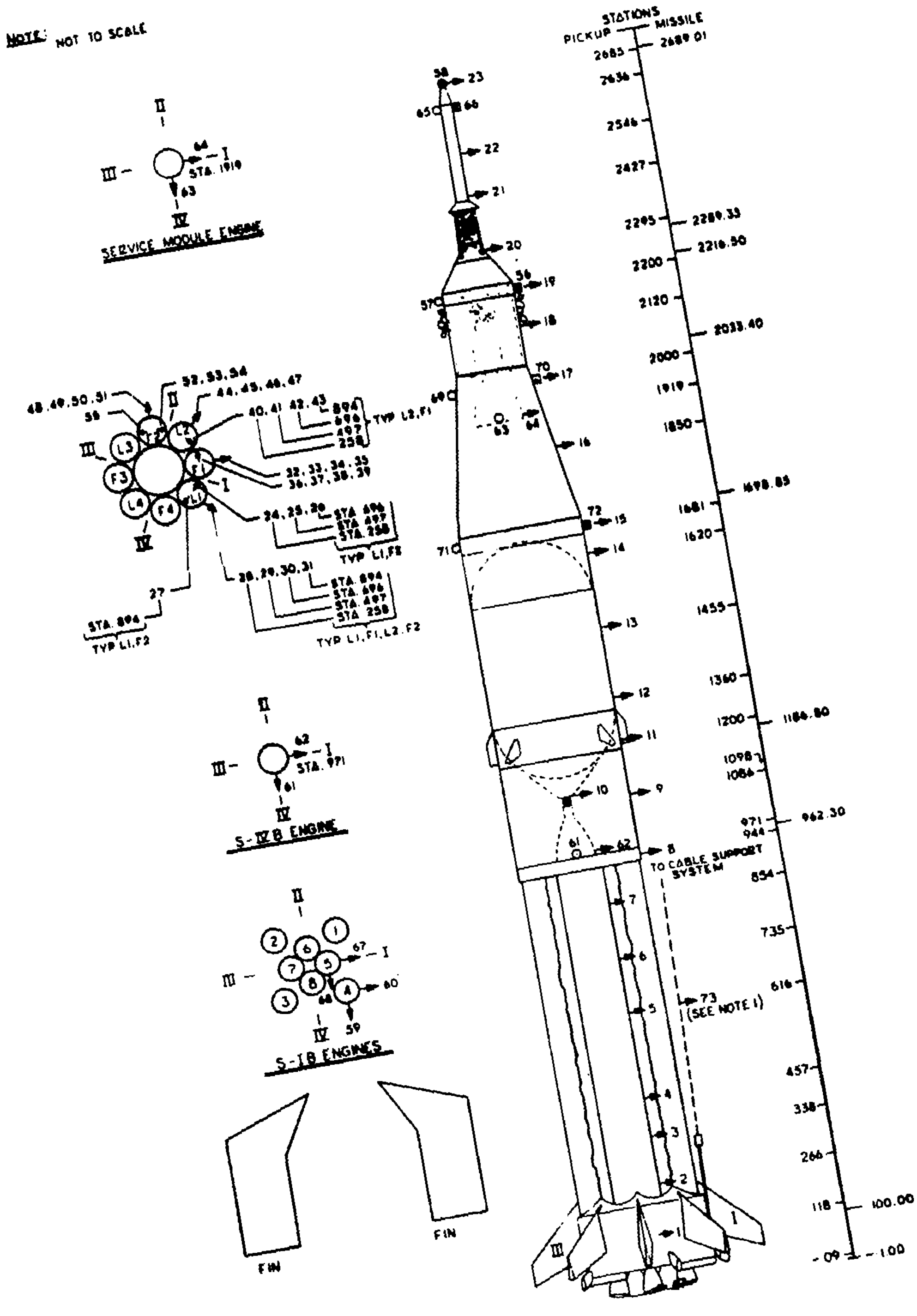


Figure 2. Block Diagram for Flight Control Testing



...lerometer Location Drawing, SA-202 Configuration (Pitch)

The decision as to how much of the filter, integrator, or signal conditioning equipment to use depends upon the particular conditions. For parameter evaluation one would use as little unevaluated vehicle equipment as possible. For final verification and compatibility checkout the ideal would be to use as much airborne equipment as possible.

3.2 STATIC TEST ARTICLES

The term "static test article" as used herein refers to a captive launch vehicle with the engines firing. This may be any configuration - a propulsion test tower, "battleship" vehicle, prototype, or a flight vehicle being tested prior to flight. The basic objective of such a test is the determination of the effect of operating environment, thrust forces, piping torques, friction, etc. upon the system performance of the control system.

The basic restriction upon such a test is the limited amount of time available. The actual firing time will be limited, therefore imposing a very tight time schedule on the test program. For this reason, discrete frequencies must be run the minimum number of cycles necessary to obtain a steady-state value, and times between frequencies must be closely regulated as to minimum time. All test programs will be run several times using non-firing engines, the basic objective being to obtain correlation between a mathematical model and both a non-firing and a firing engine system.

The control system parameters can thus be evaluated for both prelaunch checkout and flight configuration. All changes or parameter variations will normally be evaluated using the dynamic test model; the static test model will be used only for final verification.

Unstable launch vehicles require high-performance servo systems. The bandwidth of this servo may extend up to a point where secondary effects may deteriorate the performance of the system. Typical of these phenomena are: engine chugging, acoustic or engine structural vibration, coupling of control and power system, and local resonances excited by the above.

During the analysis and synthesis of the flight control system a number of assumptions are made as to which degrees of freedom are insignificant and can be omitted from the analysis. The static test provides the final check, prior to flight, of these assumptions. A typical static test program is described in References 3 and 5.

A hot firing may place certain restrictions upon the engine gimbal capability. This may be brought about either by the flame bucket, exhaust deflector, or by the loads capability of the restraining structure. In the case of the Atlas vehicle this restraint severely limited the engine gimbal capability. The engine gimbal limits are ± 5 deg, but the limit on the static test stand was ± 1 deg.

The usual mode of operation of the control tests is to apply external stimulation (ramps, steps and frequency response) to the control system. The point of input depends upon the system design and information desired. In general these signals are placed in the loop following the autopilot sensors. When possible the sensors output will be monitored open-loop.

Complete closed-loop operation of the control system is not usually possible on a static test article. The holddown mechanism usually restrains the launch vehicle so as to cause a control moment reversal. This will usually cause an instability, an oscillation limited only by flight control system limits, when the vehicle control system is completely operational in the launcher. For flight, this effect is usually eliminated by activating the autopilot after the vehicle is free of the restraining structure (Reference 6).

4/TYPICAL TEST PROGRAMS

4.1 GENERAL DISCUSSION

Once a test specimen is obtained, the test and data reduction programs must be determined. These will vary from sketches in the engineer's notebook for the earlier tests to formal programmed runs on static test articles. The emphasis will be upon the formal runs with only an overview of the possible additional testing performed on dynamic test stands.

Early test stand work is done to evaluate engine system parameters. This is necessary to finalize the basic control model and set up component and system specifications. It could also be used for combination test stand/computer simulations.

In the later phases of the program, flight control testing is conducted for the purpose of obtaining performance data to demonstrate the flight readiness of the system. In this category are the servo loop frequency tests, the servo static gain tests, and the flight programmer tests. As with dynamic test models the responses from static test articles are best used to evaluate previously derived mathematical models. Then the results are extrapolated to flight data.

Test inputs usually fall into three basic categories

a. Sinusoidal frequency inputs

b. Step inputs

c. Ramps or slowly varying inputs

Of these, frequency response inputs are almost always deliberately introduced. Steps or ramps may be introduced either by deliberate commands or by manipulation of the control system. For example, a ramp input may be obtained by allowing either the gyro or the filter integrator to drift. Once the required drift is obtained, step commands may be obtained by grounding and ungrounding the amplifier output. During hot firings, time is usually of the essence and only deliberate inputs will be used. During cold gimbaling, both types will be used.

4.1.1 OPEN LOOP. The majority of thrust vector control testing is performed with the control system operated open loop. This means that the attitude and guidance loops are not closed through the structure. The majority of testing will also make use of deliberate commands. Of the various types of commands - ramps, triangular waves, steps, and sinusoidal waves - the sinusoidal input requires the most preparation to obtain maximum output.

The problems fall into three classes

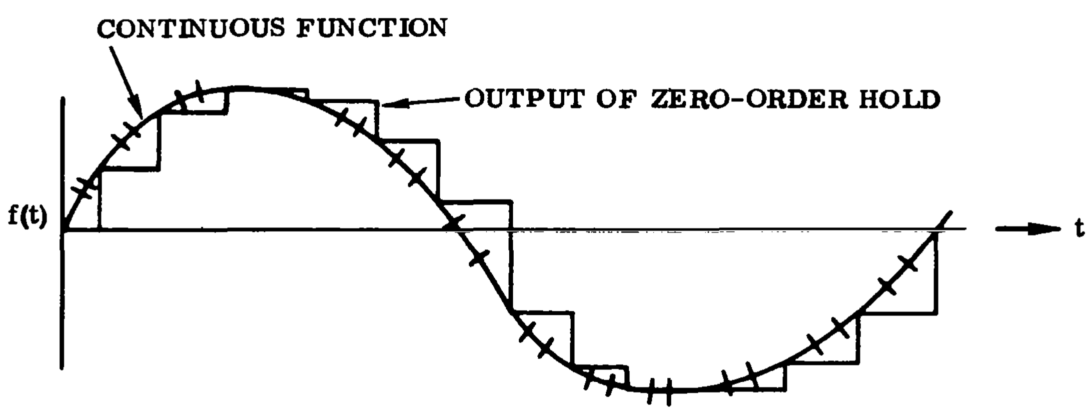
a. Making sure sufficient time is available for response to reach steady-state values.

b. Allowing sufficient time for previous response to die out.

c. Maintaining loads within reasonable limits.

Sufficient time for the response can be assured either by precomputing or by testing. When testing is performed the principle is straightforward. The magnitude of each peak is checked and compared with two or three previous peaks. When these agree within some error criterion, a steady-state response is assumed. There are two problems associated with this method. First, a disproportionate amount of time may be expended at one frequency, thereby limiting the data obtained. Second, the system may never settle down to a reasonable accuracy. This may be caused either by noise in the system or by a large nonlinearity. Nonlinearities which could cause frequency doubling, force spikes, or rectification are particularly troublesome. These amplitude comparison schemes are not normally used for hot firings. It is almost always better to run the desired number of points even if one or two do not reach required accuracy than to miss half the desired bandwidth.

Accuracy of the testing equipment must also be considered in determining the specified accuracy. In addition, when digital equipment is used the sampling error may become significant. This can be determined for any specified comparison method. If a zero order, sample and hold circuit is used, the error is determined by the following equation:



$$\epsilon = \text{Amplitude error } \% = \frac{1 - \cos(180/P)}{1} \quad 100$$

Where P = Number of samples per cycle

$$\phi = \text{Phase error} = 180/P^*$$

From Table 1 it is apparent that for low sampling rates appreciable errors can be generated. Instrumentation and recording errors must be added to the theoretical errors. As a matter of practicality, sampling rates of less than 10 samples per cycle should not be used.

Table 1. Errors in Sine Wave With Zero-Order Hold

SAMPLES PER CYCLE	MAX. AMPLITUDE ERROR	PHASE ERROR*
5	19.1%	36
10	4.9%	18
30	0.6%	6.0
50	0.2%	3.6

The pre-computation of time to reach steady state is usually done at a very basic level. The results can be verified by running the test program on the test specimen prior to hot firing. The time for the transient to die out prior to starting the next command is usually calculated in the same manner as the time to reach a steady-state response.

For this type of calculation the analyst usually refers back to basic linear analysis. A rough linear model of a system will produce a transfer function the denominator of which contains first- and second-order terms. For purposes of obtaining the decay time, only the largest first-order lag and lowest frequency second-order terms are considered.

Given a first- or second-order term, $1/(\tau s + 1)$ or $1/(s^2/\omega^2 + 2\zeta s/\omega + 1)$ and sinusoidal excitation, the transient can be considered to have the following form.

* This error applies to an artifically smoothed curve only. When using raw data the phase angle can be calculated to much greater accuracy.

$$\begin{array}{ccccccc} \text{SINE WAVE} & \times & \text{FIRST ORDER LAG} & \simeq & \text{TRANSIENT} & + & \text{STEADY STATE SINE WAVE} \\ \dfrac{1}{\left(\dfrac{s^2}{\omega_I^2}+1\right)} & \times & \dfrac{1}{(\tau s+1)} & \simeq & K_1 e^{-\left(\frac{t}{\tau}\right)} & + & \dfrac{K_2}{\left(\dfrac{s^2}{\omega_I^2}+1\right)} \end{array}$$

or for a second-order system

$$\frac{1}{\left(\dfrac{s^2}{\omega_I^2}+1\right)} \times \frac{1}{\left(\dfrac{s^2}{\omega^2}+\dfrac{2\zeta s}{\omega}+1\right)} \simeq K_1\, e^{-\zeta\omega t}\,(A) + \frac{K_2 \angle\phi}{\left(\dfrac{s^2}{\omega_I^2}+1\right)}$$

where

τ = first-order time constant

ω = second-order natural frequency

ω_I = sinusoidal input frequency

From the preceding it can be seen that the term of the form $e^{-\alpha t}$ appears in both analytical functions. Normally this term would be allowed to decay to some portion of its initial value. Three time constants ($\alpha t = 3$), a decay of the transient to 5% of original value, is the minimum. (For the system to decay to 0.1% of its original value, 6.5 time constants are required.) For nonlinear or systems where beating could occur, longer periods (two to four times as long) should be allowed. In addition to the time for the transient to decay, it is desirable to allow for at least three complete cycles at a steady-state value.

The use of a continuously varying (sweep) frequency is common practice in frequency response testing. In its most common form the frequency is smoothly varied throughout the desired range. This frequency is varied according to two basic schemes.

a. Frequency variation is linear with time

$$f = f_o\, \alpha_{LINEAR}\, t$$

b. Frequency variation is exponential with time

$$f = f_o e^{\alpha t}$$

f = Frequency as a function of time

f_o = Frequency at start of sweep

α = Parameter governing rate of sweep

The exponential sweep is greatly superior for frequency response determination as it gives an equal number of cycles in any log decrement, such as octave, of the sweep frequency. This greatly increases the overall accuracy of the plot for any given running time. Linear, or other non-exponential, schemes are used only when ease of hardware mechanization is the overriding factor.

In specifying an exponential sweep, either of the following may be used:

a. Specify a sweep of f_o to $f_{(final)}$ at X minutes* per octave.

b. Specify a sweep of f_o to $f_{(final)}$ containing Y cycles, where "Y" is the total number of cycles contained in the sweep.

The mathematics used in preparation of the sweep parameters are:

For a sweep of X minutes* per octave

$$\epsilon^{\alpha t} = 2$$

For an octave or factor of two the frequency then must equal two for the specified period of time.

$$\alpha t = \ln 2 = 0.6931471805599453$$

where t = *time in seconds required for an octave change in frequency.*

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* May be any unit of time as long as units are consistent. In practice, minutes per octave is often specified with the seconds used as the basic unit in calculations.

Thus for any t the value of α may be readily obtained. For an exponential sweep it becomes a little more bothersome.

$$f = f_o e^{\alpha t}$$

$$N = \int_{t_1}^{t_2} f_o e^{\alpha t} = \frac{f}{\alpha} e^{\alpha t} \Big|_{t_1}^{t_2}$$

for $t_1 = 0$

$$N = \frac{f_o}{\alpha} \left[e^{\alpha t_2} - 1 \right]$$

Also we know

$$f_2 = f_o e^{\alpha t_2}$$

Thus we obtain the following two relationships

$$N = \frac{f_o}{\alpha} \left[\frac{f_2}{f_o} - 1 \right] = \frac{1}{\alpha} \left[f_2 - f_o \right]$$

Also from the previous equation

$$\alpha t_2 = \ln f_2/f_o$$

As an example of the preceding, let us take a typical test input.

"Continuous sinusoidal 200-cycle sweep from 0.5 to 20 Hz"

$$200 = \frac{1}{\alpha} \left[20 - 0.5 \right]$$

$$\alpha = \frac{19.5}{200} = 0.0975$$

$$\alpha t_2 = \ln \frac{f_2}{f_o} = 3.68888$$

$$t_2 = \frac{3.68888}{0.0975} = 37.83$$

This corresponds to a sweep of 7.1 seconds per octave. This sweep rate was employed on a hot firing of the Rocketdyne MA-5 engines which had a gimbal mount resonance of around 10 to 12 Hz and gave quite useful results. There are several ways of evaluating the effect of sweep rate on frequency response (see Table 2). Rule-of-thumb would be to use a specified number of cycles depending upon accuracy required or system complexity. Lacking prior knowledge of the system, the value $\alpha = 0.1$ is a good first test value.

Table 2. Sweep Rate Characteristics

α	MINUTES/OCTAVE	COMMENTS
0.4	0.0289	Would detect gross malfunctions or changes of known systems.
0.1	0.1155	Can be used where time is limited for good comparisons with previous sweeps at identical rate.
0.0231	0.5	About as fast as normal test practice will allow.
0.01155	1	Will almost always yield AR to less than 5% error for even a very complex lightly damped structure.

There are two usual ways to check sweep rate for adequacy before final hot firings are completed. The rate may either be decreased, usually to at least half of the previous rate, or the sweep may be repeated with the frequency decreasing as a function of time. The normal effect of a sweep upon a frequency response plot is to act as a filter, i.e., attenuation of peaks and valleys plus a time delay in when they occur. Therefore, by comparing the frequency response plots obtained by sweeping both with increasing and decreasing frequency, a reasonable idea of their adequacy can be determined.

Because of the nature of the servo systems it may be necessary to change gains during the sweep. This normally occurs as a step change which introduces some transient within the system. Because of this it may be necessary to stop sweeping for a short period of time to allow a new steady-state value to be achieved. In general, such switches tend to make life complicated for the analyst and are usually avoided wherever possible.

The problem of maintaining loads within a reasonable value is somewhat outside the intent of the scope of this monograph. The structural analysis and or monitoring of loads during tests may be required to protect the test specimen and supporting structure. Even though the engine gimbaling is restricted, it may be possible to achieve large oscillations within the structure at resonance. Because of the low value of structural damping normally encountered in launch vehicles, gains at resonance (Q = steady-state amplitude out/amplitude of forcing function) of from 20 to 50 are common, with values up to 200 possible (Reference 7). Therefore, it is possible to develop large loads within the structure even at fairly low input levels.

4.1.2 CLOSED LOOP. Static testing is normally performed open-loop, the only exception occurring when a simulation involving planned vehicle motion within the stand is undertaken. This is desirable because of two reasons: first, not much information is learned from a stable run, whereas an unstable run may damage or destroy the test article and stand. Second, even if an instability occurs in the test stand it will usually have no relationship to postlaunch stability.

Tests on dynamic test models are slightly different. Without the engines firing, large amounts of energy are not present to cause destructive divergences. Also, a great deal of misconception exists, particularly in regard to control system/vehicle elastic coupling. The belief that if the vehicle is stable in launch it will be stable in flight is very persistent. Most personnel involved with a launch vehicle would like to see the closed-loop flight control system activated without the vehicle breaking into a bending oscillation.

As a result of this and the need for compatibility tests, the system may be activated in a closed-loop manner in the launcher or test stands. Such tests usually do not provide much information as to system parameters; they will thus not be discussed further.

4.1.3 COMBINED TEST SIMULATION. The test articles will be used for many types of testing. One would be combined computer/flight control system simulation. This simulation in its simplest form would have the majority of the vehicle dynamics and trajectory simulated on a computer with pieces of actual control system hardware - amplifier, integrator, engine gimbal system, etc. - substituted for their analytical models. This would provide a useful indication as to the effect of nonlinearities and second-order effects on overall system stability.

In its most complete form it could consist of a complete flight simulation using all vehicle hardware (the only exception might be guidance accelerometers) with the computer solving for vehicle dynamics only. The sensor gyros would be mounted on a three-axis flight table which would duplicate vehicle motions received from the computer. Combined test simulation is usually quite complex both as to physical setup and analytical simulation. Also, it is performed for final system verification, not for parameter verification, and therefore is outside the scope of this monograph.

4.1.4 TYPICAL TEST PROGRAM. The typical test program considered will be for a static test article. The normal control exercising of the control system for parameter evaluation will be covered in this program. When the myriad of "engineering" tests which may be performed on a dynamic test article are considered, the listing of a typical program becomes a hopeless task. A block diagram of a launch vehicle flight control system, including the test instrumentation is shown in Figure 4. From this figure it can be seen that in practice the problem is complicated by the large number of engines and by the three separate control channels. In theory this does not matter as the program can be described considering only one channel and one engine.* A typical program is given in Table 3. This program has been shortened to list one set of engines only. Both pitch and yaw are included but not roll. This is justified because all booster motion capabilities are exercised in pitch and yaw. Roll is achieved by differential motion in pitch. Special programming to exercise the electronics is not usually attempted as part of engine gimbaling during hot firing.

Dynamic response data for the engine servo loops were obtained by applying constant amplitude sinusoidal voltages and recording engine position, actuator feedback transducer outputs, and other applicable measurements. The engine position transducers are mounted on the gimbal blocks, and they monitor actual engine position if different from that of the actuator. The frequency response tests were performed "open-loop" (gyro signal amplifiers grounded). The filters (integrators) were modified in order to eliminate filtering of the test input signal frequencies over the useful bandwidth, typically (0 to 35 Hz). This permitted determining the dynamic response characteristics of the engine servo loops exclusive of the filtering circuits. The critical autopilot frequency response tests were performed under hot firing conditions. There were, however, cold (non-firing) frequency response tests performed on the engines. In general all tests are performed a minimum of three times. First, prior to the hot firing (these can be performed with various tanking levels), then during hot firing, and finally after hot firing, usually with an empty vehicle. The empty condition is also often performed prior to tanking for the hot firing.

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* The only exception being tests later in the program for coupling between channels and engines.

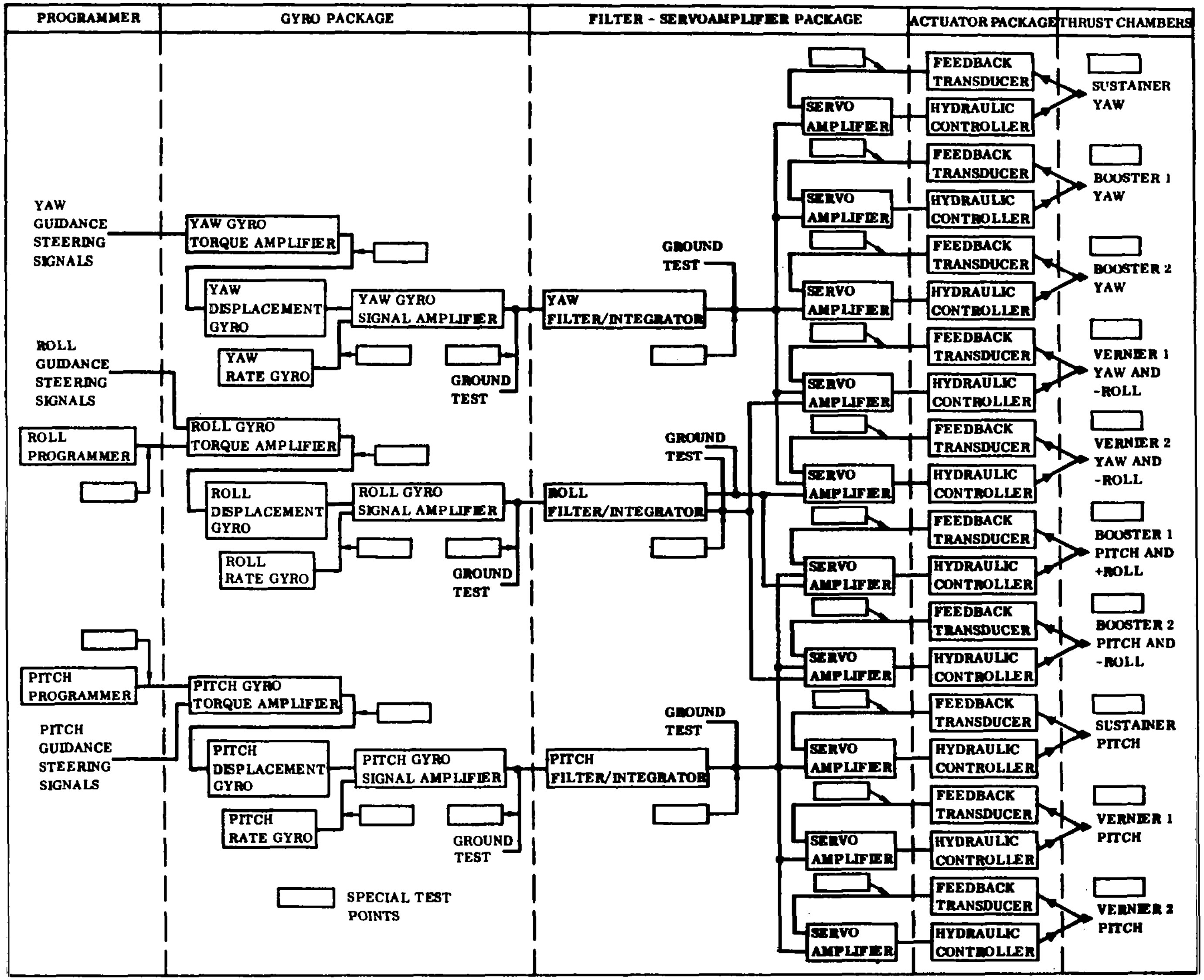

Figure 4. Launch Vehicle Flight Control System, Including Test Instrumentation

Table 3. Thrust Chamber Gimbaling Programs

The following is a list of typical gimbaling programs used during hot firing of a launch vehicle.

THRESHOLD DATA

Ramp inputs to cause ±0.25 degree deflection in pitch and yaw consecutively, at a constant rate of 0.08 deg/sec.

Ramp inputs from null to 0.8 degree in one direction, then to 2.8 degrees in the other direction, then back to null. Ramp rate from peak-to-peak is twice the rate from null-to-peak and from peak-to-null; direction of initial ramp alternated. Test performed at null-to-peak rates of 2, 4, 6, 8, 10 and 11.5 deg/sec in the pitch and yaw planes consecutively.

STATIC GAIN AND LINEARITY

Staircase inputs to cause step deflections from null to 0.25 deg, 0.50 deg, 0.75 deg, 0.50 deg, and 0.25 deg in positive pitch, then back to null; then to 0.25 deg, 0.50 deg, 0.75 deg, 0.50 deg and 0.25 deg in negative pitch, then back to null. Corresponding staircase inputs applied to repeat the same gimbaling program for positive and negative yaw deflections.

GIMBAL BEARING FRICTION

Ramp inputs from null to 1 degree in one direction, then to 1 degree in the other direction, then back to null. Ramp rate from peak-to-peak is twice the rate from null-to-peak and from peak-to-null. Direction of initial ramp alternated. Test performed at peak-to-peak rate of 2 deg/sec in the pitch plane.

DYNAMIC RESPONSE

Continuous sinusoidal 200-cycle sweep from 0.5 to 20 Hz at an input amplitude of ±1.05 volts (±1/2 deg) applied consecutively to the pitch and yaw channels.

Sinusoidal inputs (±1/2 deg) at discrete frequencies of 1, 3, 5, 7, 9, 11, 13 and 15 Hz applied consecutively to the pitch and yaw channels.

Continuous sinusoidal 40-cycle sweep from 0.5 to 15 Hz at an input amplitude of ±1.05 (±1/2 deg) applied consecutively to the pitch and yaw channels.

Sinusoidal inputs (±1/8 deg) at discrete frequencies of 1, 2, 3, 4, and 5 Hz applied to the pitch channel.

Table 3. Thrust Chamber Gimbaling Programs, Contd

DYNAMIC RESPONSE (Contd)
Continuous sinusoidal 160-cycle sweep from 0.5 to 20 Hz at an input amplitude of ±0.52 volt (±1/4 deg) applied consecutively to the pitch and yaw channels.
Sinusoidal inputs (±1/8 deg) at discrete frequencies of 1, 2, 3, 7, 9, 11, 13, 15 and 17 Hz applied to the pitch and yaw channels.
Sinusoidal inputs (±1/2 deg) at discrete frequencies of 10, 11, 12, 13, 16, 17, 23, 24, 25, and 27 Hz applied to the yaw channel, and discrete frequencies of 13, and 23 Hz applied to the pitch channel.

All responses specified in Table 3 should be self-explanatory with the exception of the high rate ramp. For clarity this is plotted as Figure 5.

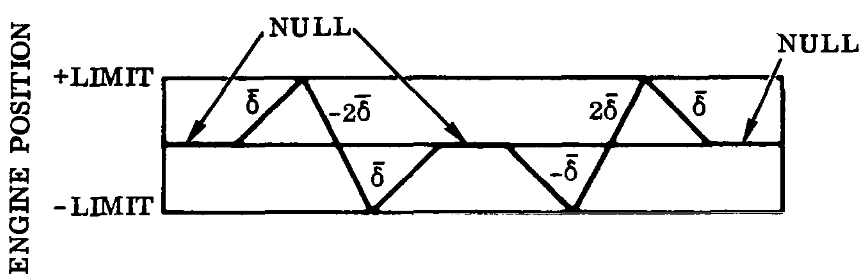

Figure 5. High Rate Ramp Input

4.2 DATA OBTAINABLE

The types of data which can be obtained from full-scale test stands will be discussed. This is intended primarily as a pre-setup guide for types of tests and data which may be of interest.

The basic area of interest to the control analysis is frequency response, gain, and phase of the main thrust vectoring system. The various parameters - feedback transducer, engine or vane position, servo output, hydraulic pressure, electrical voltage and response of structure - may be evaluated. The feedback transducer or engine output is usually considered to be the primary output with the other variables usually classed as secondary effects and presented as a group with cross-coupling terms.

Dynamic cross-coupling can occur through either inertial or structural unbalance or a combination of both and can occur in almost any portion of the system. This is usually uncovered first by testing as it is not normally desired or designed into the control system. As full-scale testing may be the first time that possible cross-coupling terms will be observed, all output instrumentation should be monitored for step input and frequency response testing.

Frequency response testing is used for direct parameter evaluation for model correlation. The parameters which may be altered to obtain this correlation cannot be rigorously defined. The normal procedure is to vary the parameters within their analytically estimated limits. Those parameters which are defined with the least confidence are varied first. When reasonable parameter values do not produce a fit then an analytical review of all factors considered within the model must be performed.

The next class of inputs are the static quasi-dynamic measurements made for direct parameter evaluation. These include static gain, linearity, threshold, dynamic gain *, static limits, clearance flow limits, restraint torques, friction, leakage flow or steady-state current, apparent stiffness**, dead-zone*** and cross-coupling.

The cross-coupling consists of motion in one axis as a result of commands in another. The cross-coupling is usually grouped into two categories, static and dynamic. The static terms may be either intentional (actuators positioned off-axis) or unintentional (such as would be caused by misalignments or tolerance buildup). These static terms can be evaluated on almost any test setup provided adequate instrumentation is available. Optical instrumentation is quite often used for static cross-coupling as well as linearity.

The effect of the expected duty cycle is usually first evaluated on dynamic test models. For these tests, external commands are used to exercise the system equivalent to worst expected flight time and conditions. The ability of the system to operate with pressure, voltage, temperature, and performance limits is evaluated by these tests. While extremely important to the overall system performance, duty cycle tests do not usually provide parameter evaluation and so will not be discussed further in this monograph.

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* Dynamic gain consists of the steady-state open loop gain of a system. Normally deg/sec per volt input or some similar units.

** On many systems considerable information can be gained by measuring the steady-state spring constant of a closed loop servo system in responding to disturbance forces or torques.

*** Dead-zone is considered to differ from threshold in that it is a system parameter. A simplified model will often contain a position of velocity dead-zone.

The last class of data obtainable is elastic properties. These are body and stand resonances, control system and mount resonances, and transfer functions between control force generators and vehicle sensors. In addition to verifying the analytical properties of the vehicle, unexpected modes will be uncovered at this time. These will be modes involving coupling mechanisms not considered in the basic model, such as skin modes.

5/DATA REDUCTION AND APPLICATIONS

5.1 STATIC GAIN AND LINEARITY

Data reduction is accomplished by hand, by specific data reduction equipment, and by modern computing techniques. The methods to be employed depend on the magnitude of testing to be done, the availability of equipment, and economic considerations. The methods presented here are adaptable to either hand or machine reduction.

The first tests usually evaluate what is referred to as static gain and linearity. These terms relate the actual motion of the engine bell, or the output of a sensor or amplifier versus its input. In its simplest form the relationship results in a plot like Figure 6. The gain consists of the slope of the line, and the system is linear if all points fall on a straight line.

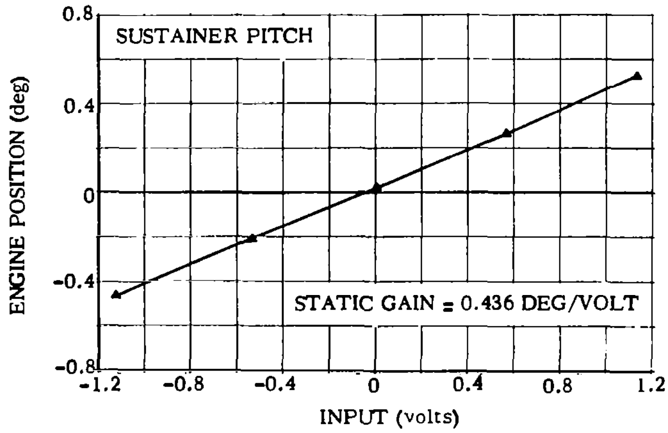


Figure 6. Typical Static Gain Plot

For acceptance testing, the value of gain, linearity, dead-zone, and hysteresis are subjects for contract definition. For the control analyst the problem is somewhat simpler. The usual process is to plot the points and draw a straight line through them. For more precise measurements a least-squares curve fit may be calculated. For preliminary model evaluation a linear approximation, usually least-square or with absolute error, to the gain will be used. For more complicated analysis, the following may be used:

a. Change in effective gain (slope of line) with amplitude of input. This is basically a describing function approach.

b. An analytical model, sine wave, square, square root or limit of the test curve.

c. A function generator to generate a function which will pass through all the test points on the test curve.

Hysteresis, backlash and deadband are evaluated by obtaining a line for both increasing and decreasing direction of the input voltage (see Figure 7).

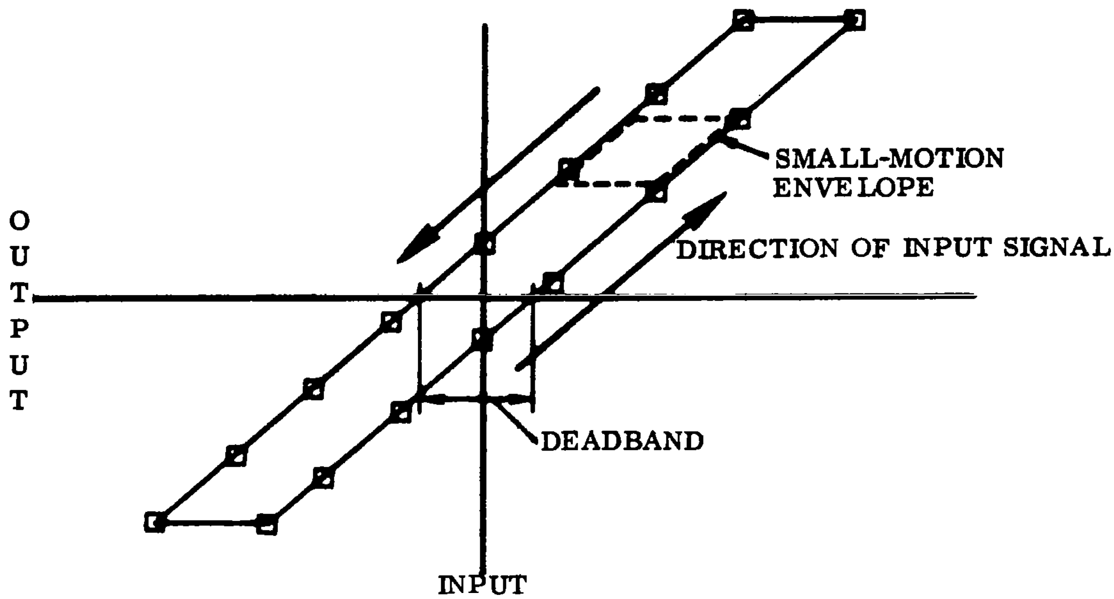


Figure 7. Typical Backlash or Velocity Dead-zone

The numerous parameters to be evaluated are model-dependent. The best use of results will be to attempt to verify constants within a particular model. A complete discussion of model development and characteristics can be found in References 8 and 9. Because of the use to which these terms are put, computer programs are not normally used to find exact values of linearity, backlash, or dead-zone. Computer programs may be used in scaling, curve fitting, and statistical (regression) analysis.

Along with static gain and linearity such items as friction and effective restraining spring force are determined.

This data is required for analysis of hydraulic requirements. In addition, the tests provide data in support of analog computer simulation studies and analytical describing functions of autopilot servo-loop nonlinearities. The formulas for determining coefficients of viscous and Coulomb friction for the booster and sustainer engines are derived from the equation of motion of the thrust chambers.

$$AR\,\Delta P = I\ddot{\delta} + B_v\,\dot{\delta} + K\delta + B_c\,\frac{\dot{\delta}}{|\dot{\delta}|} \tag{1}$$

where

AR = piston area times the moment arm

ΔP = differential pressure across the hydraulic actuator piston

I = moment of inertia of the engine with respect to the axis of rotation

δ = angular deflection of the engine

$\dot{\delta}$ = angular velocity of the engine

$\ddot{\delta}$ = angular acceleration of the engine

B_v = viscous friction coefficient

B_c = Coulomb friction coefficient

K = effective spring constant

For typical test programs the friction data is reduced from responses to ramp inputs where

$$\dot{\delta} = \text{constant}$$

and

$$\frac{d\,\dot{\delta}}{d\,t} = \ddot{\delta} = 0$$

If ΔP is measured at a point on the ramp deflection corresponding to engine null ($\delta = 0$), then Equation (1) simplifies to

$$AR\,\Delta P = B_v\,\dot{\delta} + B_c\,\frac{\dot{\delta}}{|\dot{\delta}|}$$

Dividing by AR, ΔP becomes a function of $\dot{\delta}$ in the form

$$\Delta P = \frac{B_v}{AR}\,\dot{\delta} + \frac{B_c}{AR} + \frac{\dot{\delta}}{|\dot{\delta}|}$$

For a slope-intercept form of a straight-line equation

$$y = mx + b$$

where

$$y = \Delta P$$

$$x = \dot{\delta}$$

$$m = \frac{B_v}{AR} = \text{slope of } \Delta P \text{ vs } \dot{\delta} \text{ plot}$$

$$b = \frac{B_c}{AR}\,\frac{\dot{\delta}}{|\dot{\delta}|} = \Delta P \text{ axis intercept of } \Delta P \text{ vs } \dot{\delta} \text{ plot}$$

The viscous friction coefficient, B_v, is computed from the slope of the ΔP vs $\dot{\delta}$ plot. The slope of this plot is given by

$$B_v = AR\,\frac{\Delta(\Delta P)}{\Delta\dot{\delta}}$$

The Coulomb friction coefficient, B_c, is obtained from the intersection of the ΔP vs $\dot{\delta}$ plot with the ΔP axis.

$$B_c = AR\,\Delta P$$

Typical plots of MA-5 engine system gimbal friction are shown in Figure 8.

5.2 FREQUENCY RESPONSE

The majority of testing time and data reduction are spent on frequency response testing. For "quick look" evaluation, the following methods were used in computing the required parameters:

a. Gain was computed as follows:

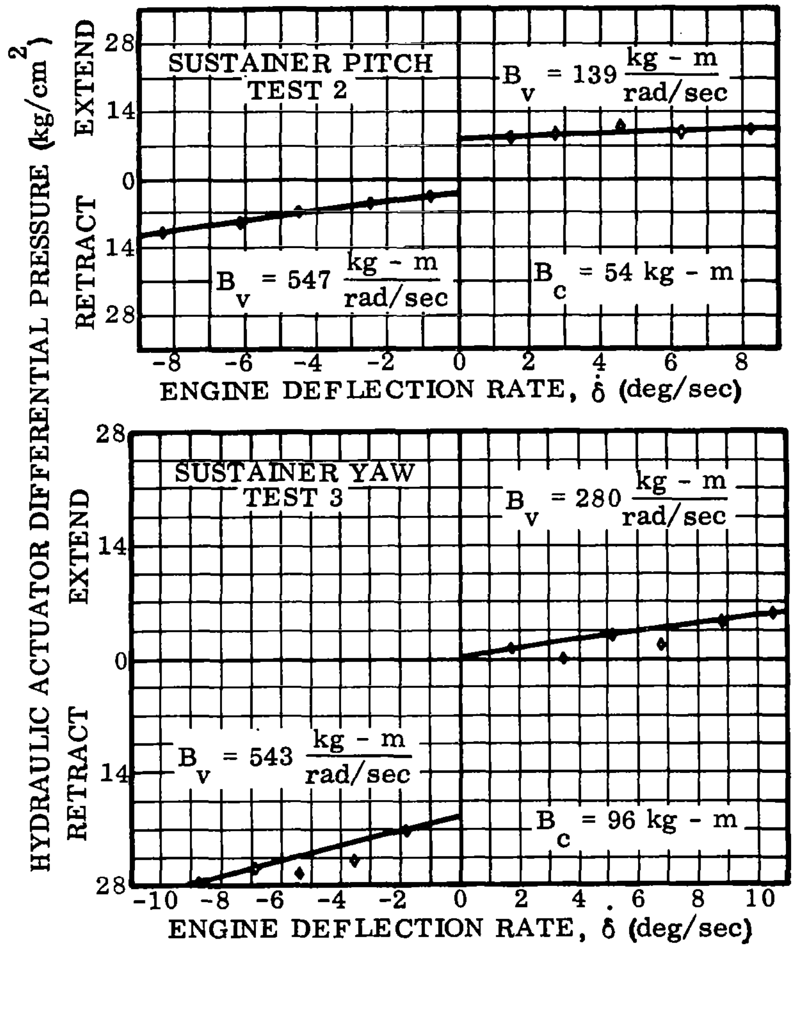


Figure 8. Coulomb and Viscous Friction of a Typical Space Booster

$$\text{Gain in db} = 20 \log \frac{\text{output}}{\text{input}}$$

where: output = output transducer or feedback voltage

input = input signal voltage

b. Phase lag was computed as follows:

$$\phi = f \times 360^{\circ} \times \Delta T$$

where: ϕ = phase lag, degrees

f = frequency, Hz

ΔT = time lag, seconds, measured at null crossing

For a sweep frequency input the process is repeated a number of times throughout the sweep, and the reduction and plotting are handled as though the frequency were that of the mid-point of the input cycle.

For more precise or automated procedures several frequency response analyzers are available. Devices may yield either amplitude or phase components. The following is a brief outline of a scheme usually employed.

For automated frequency-response calculations, Fourier transform methods are employed. In this system the output is compared with the input for in-phase and out-of-phase components. The following relationship holds.

Apply a sinusoidal input to the system to be analyzed

$$\theta_C = A \sin \omega t$$

which has an output θ_O

$$\theta_C \;—\; \boxed{\begin{matrix}\text{CONTROL}\\ \text{SYSTEM}\end{matrix}} \;—\; \theta_O$$

The in-phase Fourier components are then calculated by an integration of the product

$$\phi_I = \int_0^T (\theta_C \cdot \theta_O)\, dt = \int_0^T A \sin \omega \cdot \theta_O \, dt$$

A similar procedure is used for the out-of-phase component.

$$\phi_O = \int_0^T A \cos \omega t \cdot \theta_O \, dt$$

This integration can be done in numerous ways: electrical, mechanical, or numerical. In any case the average value must be obtained, either by filtering the output of the integrators, or by numerical integration for one cycle only and dividing by the interval.

Thus the in-phase component is

$$B_I = \frac{2\phi_I}{A}$$

and the out-of-phase component

$$B_O = \frac{2\phi_O}{A}$$

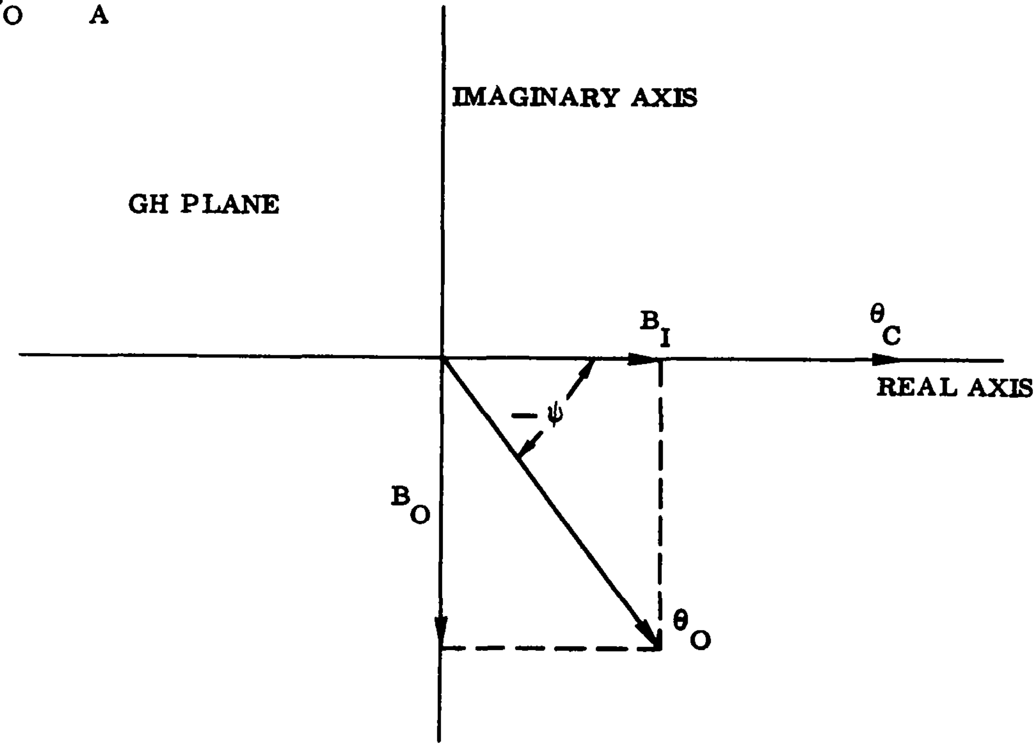


Figure 9. Component Plots

The output can be calculated from basic geometry with the output magnitude

$$\theta_O = \sqrt{A^2 + B^2}$$

and the phase angle

$$\psi = \tan^{-1} \frac{B_O}{B_I}$$

This analysis has the advantage of giving a better value for nonlinear systems than straight reduction from recorder traces. This occurs because the Fourier analysis removes the errors caused by harmonic content.

The deviation of frequency response analysis by Fourier methods is devised in Reference 9 and in Volume 1 of Reference 11.

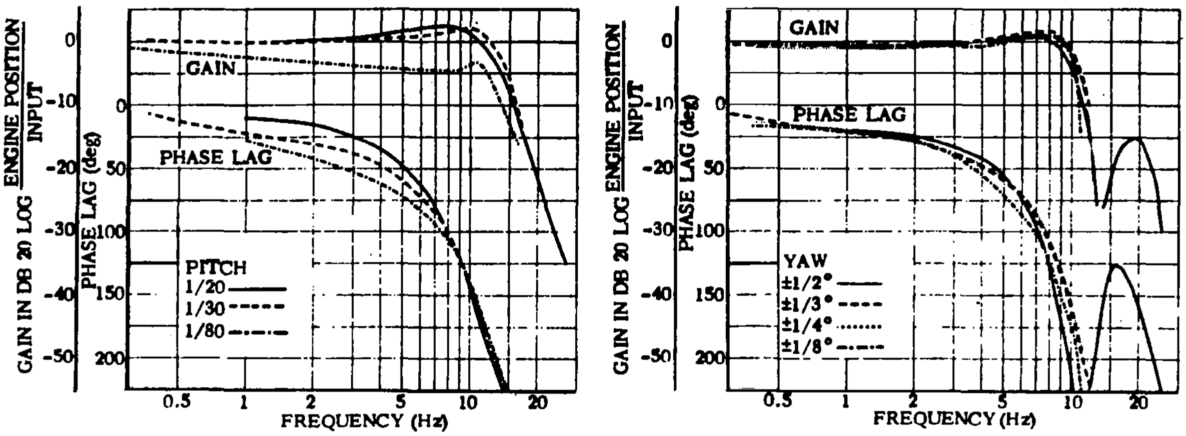
For sweep frequency input, the amplitude ratio and phase shift are calculated either continuously by a frequency-response analyzer or at particular points by handling them as though they were discrete points. The spectral analyzer does have one disadvantage on analysis of sweep inputs. This arises because the filtering on the integrals will cause appreciable lag in the output and thus result in increased error. Because of this it is quite often necessary to sweep at one-half to one-quarter the usual rate to get adequate results when a frequency analyzer is used.

Typical response plots showing the effect of mounting stiffness upon frequency response are shown in Figure 10a, with Figure 10b showing the differences in frequency response caused by firing of the engines.

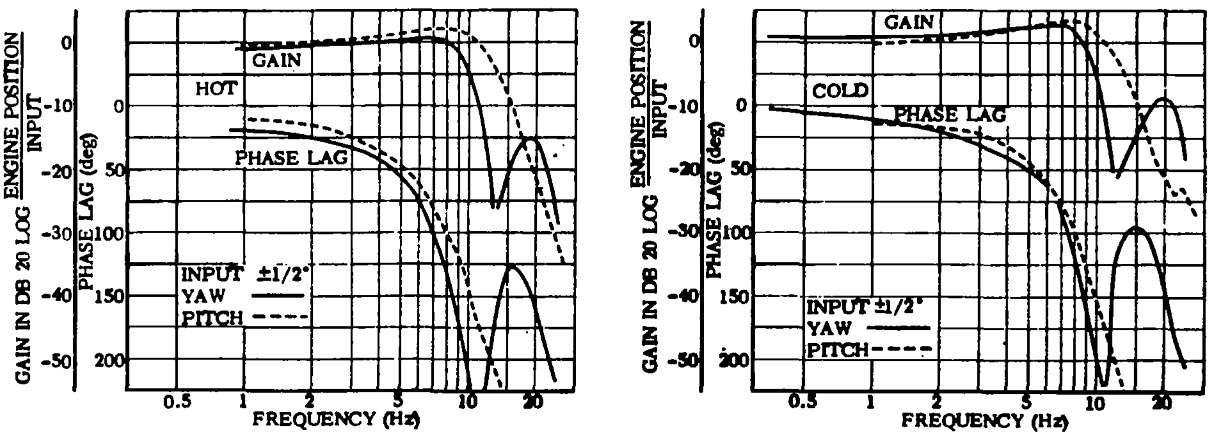
5.3 TRANSIENT RESPONSE

The basic characteristics of a flight control system may be obtained by observing its transient response. For these responses the basic inputs: pulse, step, and ramp are most commonly used.

The first tests would evaluate the error coefficients. Ideally this would be done first with the system feedback loop open, to give a direct indication of the forward loop transfer function. These inputs can be applied at many points within the forward loop for the various components and combinations of components within the control system. The basic principles used to evaluate the system parameters based on transient response are the same for all tests.



a. Effect of Mounting Stiffness

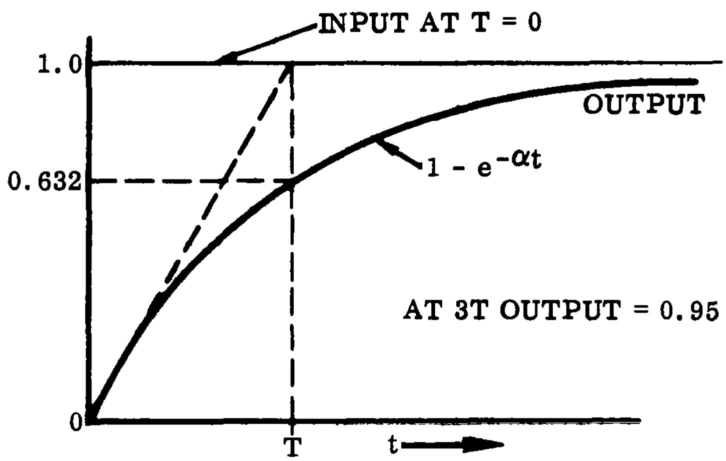


b. Effect of Engine Firing

Figure 10. Typical Response Plots

The data reduction can be approached in two methods: quick look or computational methods. With quick-look methods one tries to identify the characteristics of the basic roots of the transfer function from the response. The basic characteristics of the response of a first- and second-order system to a step response are summarized briefly below. For a more complete discussion the reader should refer to a servomechanism text book, such as References 8, 10, or 11.

A system with a first-order transfer function ($1/\tau_s + 1$) responds to a step input with an output transient of the form $1 - e^{-\alpha t}$ (see sketch below).



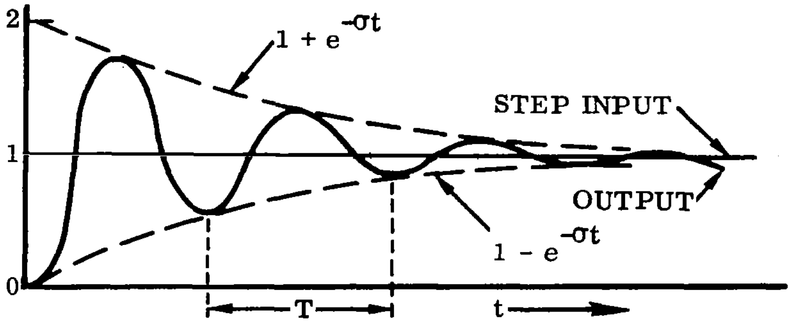
The value of time that makes the exponent of e equal to -1 is called the time constant T. Thus

$$-\alpha T = -1$$

$$T = 1/\alpha$$

The same rate of convergence to final value also can be used to calculate the damping of a second-order system. This type of system has a transfer function, $\dfrac{1}{\dfrac{s^2}{\omega^2} + \dfrac{2\zeta s}{\omega} + 1}$,

and responds with a transient of the form, $1 - e^{-\sigma t} \sin (\omega_d t + \phi)$



where the period $P = \dfrac{6.28}{\omega_d}$

and $\sigma = \zeta\omega$

$$\omega_d = \omega\sqrt{1 - \zeta^2}$$

This equation can usually be solved directly, first for ζ using the approximation $\omega = \omega_d$ and then for ω. If ζ is large, an iterative process may be required.

Where the damping ratio falls within the normal servo range $\zeta = 0.1$ to 0.6 it can be approximated quite accurately by observing the overshoot of the first peak (see Figure 11).

For a third-order system a satisfactory approximation to the characteristics can usually be achieved by graphically separating the first-order time lag from the oscillatory portion. Figure 12 shows a typical third-order response and its components.

$$1 - 0.33e^{-\alpha t} - 0.66e^{-2\zeta\omega t} \sin\left(\omega\sqrt{1 - \zeta^2}\, t + \psi\right)$$

The parameters are normalized so that

$$\alpha = 1$$

which gives

$$\zeta = 0.3 \quad \omega = 3.14 \quad \psi = -95^\circ$$

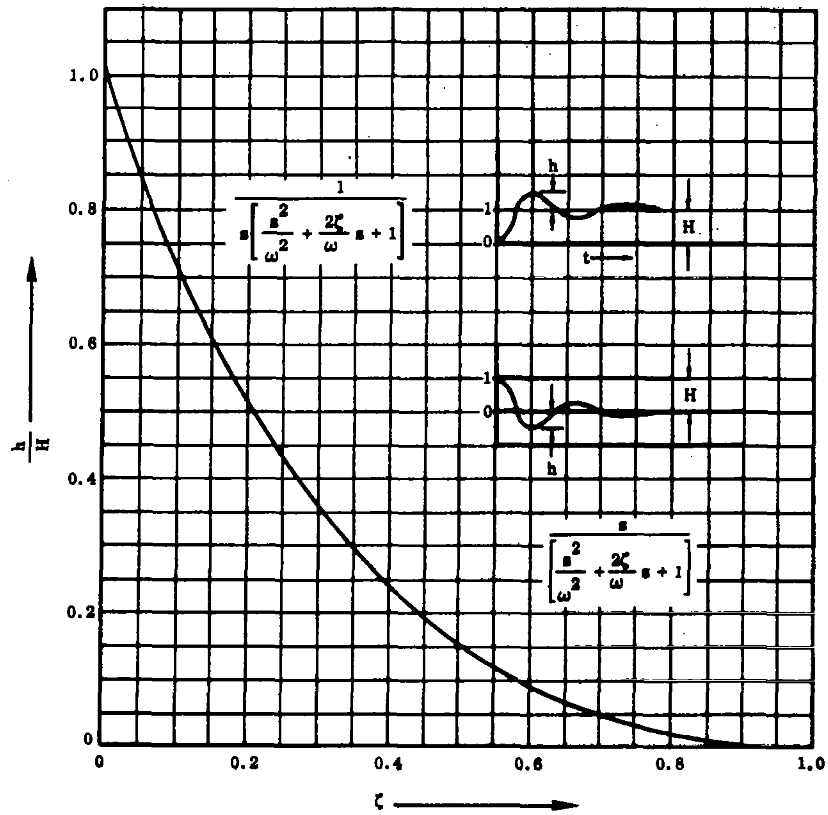


Figure 11. Determination of Damping Coefficients for Second-Order Systems from Response Curves

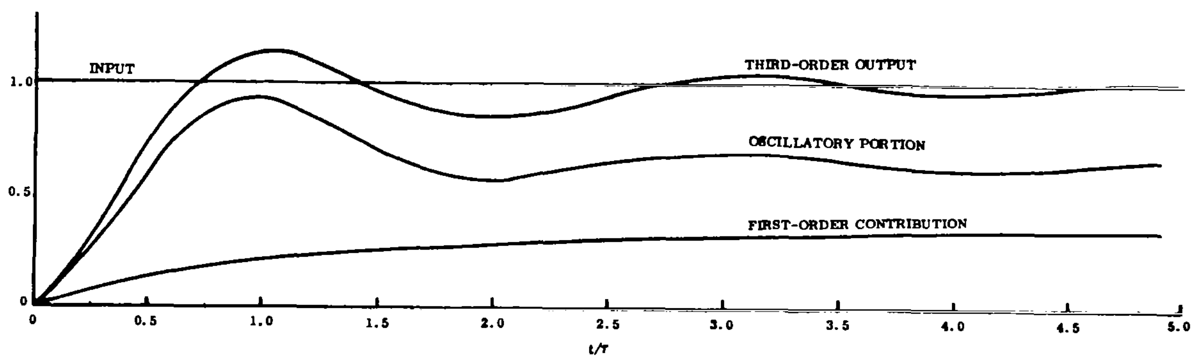


Figure 12. Typical Third-Order Response and its Components

While the quick-look approach is fine for preliminary control studies, the transient response does have potential for more accurate and detailed evaluation. Theoretically it would be possible to completely describe any linear system from its step response. In practice, the higher order effects will be lost because of noise, nonlinearities, and instrumentation inaccuracy.

Transient analysis can be approached in several ways. The most common is Fourier analysis of the response. This is covered in Chapters 25, 26 and 27 of Volume II of Reference 11. Because of the myriad of practical problems associated with analysis of step response, the more complex computer analysis techniques are not used to a great degree. This is particularly true where the system is extremely nonlinear. What has been said previously about step inputs also holds for pulse, triangular, versine and other short inputs. These inputs are quite often used for transient response where it is desirable to attenuate some frequencies and accentuate others. The response of second-order system to pulse inputs is discussed in some detail in Chapter 8 of Reference 12. Although complex analyses are avoided, step responses are plotted and used as overlays for comparison with analog or digital simulations.

5.4 ADDITIONAL AVAILABLE OUTPUT

5.4.1 CONTROL CONSIDERATIONS. The exercising of the engines, particularly sinusoidal, provides the opportunity to check the airframe transfer function. Looking at Figure 3 we see that the airframe can be instrumented, (flight control gyros or accelerometers being the most common sensors) , and the response of each sensor evaluated for all inputs. The outputs are usually plotted as though they were normal frequency response plots as outlined in Section 5.2.

This form of testing is particularly valuable for higher modes, 3rd or above, and modes involving other than pure lateral bending such as skin modes, breathing modes, or coupled lateral/longitudinal modes. These modes are normally not greatly affected by the hold-down restraint as are rigid body and the first few bending modes. This type of test closely parallels that done in full-scale testing for mode determination, Reference 4.

Static testing presents one chance to monitor the operation of all systems under an environment which is closer to flight environment, particularly thermal (cryogenic), high frequency airframe vibrational, and acoustic, than is possible with methods discussed previously. Therefore, sensors, servos - in fact all flight control components - must be analyzed to determine if this environment has any deleterious effect on the system operation. Particular attention must be paid to saturation or partial saturation of the sensors or servos. Such saturation, noise or vibration occupying a significant portion of the component capability, affects the system by effectively reducing the system capacity by the amount of the noise and can be serious. Also, this environment will tend to introduce biases into the output of the sensors or servos, which may cause actual errors in operation and affect performance.

5.4.2 AIRFRAME VIBRATION. Airframe vibration will usually be recorded during all static testing. The handling of this data is discussed in Reference 12. There may be a requirement for quick analyses to verify that the general vibration levels will not affect servo performance. For this the following quick-look methods are employed. The vibration data acceleration will be reduced and plotted, usually by analog methods, in the form of power density spectra. The rms value of the acceleration between two frequencies is derived by taking the square root of the integral of the power density spectra from the lower to the higher frequency. The following is a brief presentation of the method.

$$g(\text{rms})\Big|_{f_1}^{f_2} = \int_{f_1}^{f_2} \sqrt{\phi\,(f)df}$$

where:

$$\phi\,(f) = g^2/\text{Hz} \quad \text{(power density)}$$

The affect of this acceleration on a system with a natural frequency ω and an apparent damping ζ can be approximated by a sine wave of amplitude A at frequency ω where

$$A = 1.414\ G\ (\text{rms})\Big|_{f_1}^{f_2}$$

The values to be used for f_1 and f_2 are derived by using the half-power point of the system to be checked. For systems with low damping the half power point occurs within $\pm\zeta\,\omega$ of the natural frequency. Therefore, for a first approximation to the effect of airframe vibration on the control system one would apply a sine wave with frequency ω and amplitude A where:

$$A = 1.414 \int_{\omega(1-\zeta)}^{\omega(1+\zeta)} \sqrt{\phi\,(f)\,df}$$

When a more precise evaluation is required, a complete treatise on the effect of random inputs on systems, such as Reference 9 or 12, should be consulted.

A portion of a launch vehicle's operational disturbance will be in the form of acoustic energy. When large amounts of acoustic energy occur, it may cause saturation or partial saturation within microphonic, output reflecting acoustic input, components. Microphonics notwithstanding, the major launch vehicle problem with sound is one of fatigue, as servo bandwidth is always limited to frequencies lower than those generated by normal acoustic sources. When the acoustic disturbance is predominantly at one frequency, its effect can be quickly evaluated by changing the sound pressure level, usually in decibels (db), to a disturbing force. The force is calculated simply as pressure times the dynamic pressure and is applied as an input at the major frequency. Decibel levels are generally based on the sound pressure reference of 0.0002 dyne/cm^2 (P_o); the pressure for the threshold of hearing is approximately 1,000 Hz. Sound pressure levels can be derived from decibels by the following relationship:

$$\text{sound pressure level} = 20 \log_{10} \frac{P(rms)}{P_o}, \text{ decibels (db)} ;$$

as a reference point, 10^6 d/cm^2 $\approx$ one atmosphere

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